Computations of Scattering Lengths in $nnpp$ System within Cluster Reduction Method for Yakubovsky Equations*

S.L. Yakovlev† I.N. Filikhin

February 9, 2008

Department of Mathematical and Computational Physics, St.Petersburg State University, Ulyanovskaya Str. 1, 198904 St.Petersburg, Russia

1 INTRODUCTION

The elastic and rearrangement processes in the four nucleon system with two clusters in the initial and final states can be treated adequately in framework of Yakubovsky differential equations (YDE) approach. Unfortunately, a direct application of YDE to the scattering problem requires huge computer resources. It is why, new methods allowing a reduction of comlexity of YDE are of interests from the point of view of practical calculations. In papers [1], the authors proposed a method which reduces the YDE to the equations for the functions describing the relative motions of clusters. This method of cluster reduction (CRM) was successfully applied to calculations of n–^{3}H scattering in [1]. It is worth to note that the exact four nucleon calculations of n–^{3}H scattering were performed on a personal computer what can characterize an efficiency of CRM.

In this report we give a sketch of CRM and our recent results of calculations of scattering lengths in the four nucleon system.

2 CRM FORMALISM

In this section we present a brief review of CRM for the low energy scattering problem in the system of four neutral particles. The more comprehensive description can

*Contribution to XV International Conference on Few-Body problem in Physics, Groningen, the Netherlands, 22–26 July 1997

†E-mail: yakovlev@snoopy.phys.spbu.ru

be found in the papers [1]. The generalization on the case of charged particles is straightforward following the results of [2].

The starting point is YDE approach. In framework of YDE the four body wave function should be decomposed into components $\Psi_{a_3a_2}$ in one to one correspondence to the all chains of three cluster (a_3) and two cluster (a_2) partitions. The components $\Psi_{a_3a_2}$ obey the YDE [3]

$$(H_0 + V_{a_3} - E)\Psi_{a_3a_2} + V_{a_3} \sum_{(c_3 \neq a_3) \subset a_2} \Psi_{c_3a_2} = -V_{a_3} \sum_{d_2 \neq a_2} \sum_{(d_3 \neq a_3) \subset a_2} \Psi_{d_3d_2} \qquad (1)$$

For the two clusters collisions the YDE admit a further reduction. Let

$$H_0 = T_{a_2} + T^{a_2}$$

be the separation of kinetic energy operator in the intrinsic T_{a_2} with respect to the clusters of a_2 part and the kinetic energy T^{a_2} of the relative motion of a_2 clusters. The cluster reduction procedure consists in expanding of the components $\Psi_{a_3a_2}$ along the basis of the solutions of Faddeev equations (FE) for subsystems of the partition a_2

$$(T_{a_2} + V_{a_3})\psi^{a_3}_{a_2,k} + V_{a_3} \sum_{(c_3 \neq a_3) \subset a_2} \psi^{c_3}_{a_2,k} = \varepsilon^k_{a_2} \psi^{a_3}_{a_2,k}.$$

The actual expansion has the form

$$\Psi_{a_3a_2} = \sum_{k \geq 0} \psi^{a_3}_{a_2,k}(\mathbf{x}_{a_2}) F^k_{a_2}(\mathbf{z}_{a_2}) \qquad (2)$$

Here the unknown amplitudes $F^k_{a_2}(\mathbf{z}_{a_2})$ depend only on the relative position vector $\mathbf{z}_{a_2}$ between the clusters of the partition a_2. The basis of the solutions of FE is complete but not the orthogonal one [4] due to not Hermitness of FE. The biorthogonal basis is formed by the solutions of conjugated FE

$$(T_{a_2} + V_{a_3})\phi^{a_3}_{a_2,k} + \sum_{(c_3 \neq a_3) \subset a_2} V_{c_3} \phi^{c_3}_{a_2,k} = \varepsilon^k_{a_2} \phi^{a_3}_{a_2,k}.$$

Introducing expansion (2) into YDE (1) and projecting onto the elements of biorthogonal basis $\{\phi^{a_3}_{a_2,k}\}$ lead to the resulting reduced YDE [1] for $F^k_{a_2}(\mathbf{z}_{a_2})$:

$$(T^{a_2} - E + \varepsilon^k_{a_2}) F^k_{a_2} = - \sum_{a_3 \subset a_2} \langle \phi^{a_3}_{a_2,k} | V_{a_3} \sum_{d_2 \neq a_2} \sum_{(d_3 \neq a_3) \subset a_2} \sum_{l \geq 0} \psi^{d_3}_{d_2,l} F^l_{d_2} \rangle. \qquad (3)$$

Here the brackets $\langle . | . \rangle$ mean the integration over $\mathbf{x}_{a_2}$. The asymptotic boundary conditions for $F^k_{a_2}(\mathbf{z}_{a_2})$ have the following *two body* form as $|\mathbf{z}_{a_2}| \to \infty$:

$$F^k_{a_2}(\mathbf{z}_{a_2}) \sim \delta_{k0} [\delta_{a_2b_2} \exp i(\mathbf{p}_{a_2}, \mathbf{z}_{a_2}) + \mathcal{A}_{a_2b_2} \frac{\exp i\sqrt{E - \varepsilon^0_{a_2}} |\mathbf{z}_{a_2}|}{|\mathbf{z}_{a_2}|}], \qquad (4)$$

where it is implied that the subsystems of partitions a_2 have only one bound state with $\varepsilon^0_{a_2}$ being the respective binding energy. The subscript b_2 corresponds to the initial state and $\mathbf{p}_{a_2}$ is the conjugated to $\mathbf{z}_{a_2}$ momentum.

3 RESULTS of CALCULATIONS of SCATTERING LENGTHS in FOUR NUCLEONS SYSTEM

In this section we present the results of calculations of channels scattering lengths in $N-NNN$ system (without Coulomb interactions) for the states with the total isospin $T=0$, and results for scattering lengths in d–d, n–^{3}He and p–^{3}H systems. The MT I-III potential model was used to describe the N–N interaction.

In the first case the total spin S and isospin T are the integrals of motion, so that the scattering lengths A_{ST} can be introduced. The results of our calculations together with data of other authors are collected in the Table 1.

Table 1. Results of calculations of A_{ST} for $S=0,1$, $T=0$ states.

Refs.	A_{10} fm	A_{00} fm
This work	2.8	14.7
[5]	3.013	12.317
[6]	3.09	14.95
[7]	2.9	8.1
[8]	3.25	14.75

The modified version of equations (3) followed from equations [2] was used for computations of scattering lengths in the *nnpp* system taking into account the Coulomb interaction between protons. The d– d and n–^{3}He, p–^{3}H scattering lengths corresponding to the values of total spin $S=0,2$ for d–d scattering and $S=0,1$ for n–^{3}He and p–^{3}H scattering are collected in Tables 2,3 and 4.

Table 2. d–d scattering lengths for $S = 0$ and $S = 2$ states.

Refs.	$A_{d-d}(S = 0)$ fm	$A_{d-d}(S = 2)$ fm
This work	10.2 - 0.2 i	7.5

Table 3. p–^{3}H and n–^{3}He scattering lengths for $S = 0$ state.

Refs.	A_{p-^3H} fm.	A_{n-^3He} fm.
This work	-22.6	7.5 - 4.2 i
[9]	-21	7.16 - 3.9 i
[7]	4.2	6.05 - 0.72 i
Exp. [10]	-	6.53 ($\pm$ 0.32) - 4.445 ($\pm$ 0.003) i

Table 4. p-^{3}H and n-^{3}He scattering lengths for $S = 1$ state.

Refs.	A_{p-^3H} fm	A_{n-^3He} fm
This work	0.5	2.9 - 0.0 i
[7]	-	4.25 + 0.005 i

The modified equations (3) were used to calculate the 0^+ resonance state of ^{4}He. The results of calculations are presented in the Table 5.

Table 5. Results of calculations for the energy $E = E_r + i\frac{\Gamma}{2}$ of 0^+ resonance state of ^{4}He.

Refs.	E_r MeV.	Γ MeV.
This work	0.15	0.3
[9]	0.12	0.26
Exp. [11]	0.25	0.3

4 ACKNOWLEDGEMENTS

This work was partially supported by the Russian Foundation for Basic Research (Project No. 96-0217031). One of the authors (S.L.Y.) is grateful to Prof. L.P. Kok Institute for Theoretical Physics University of Groningen for the kind invitation to visit Groningen University during the period of Few Body XV Conference.

References

[1] S.L. Yakovlev, I.N. Filikhin, Yad. Fiz. **58** (1995) 817; Yad. Fiz. (1997) to be appeared

[2] S.P. Merkuriev, S.L. Yakovlev, C. Gignoux, Nucl. Phys. **A431** (1984) 125

[3] S.P. Merkuriev, S.L. Yakovlev, Yad. Fiz. **39** (1984) 1580

[4] S.L. Yakovlev, Theor. Math. Phys. **102** (1995) 323; **107** (1996) 513

[5] V.F. Kharchenko, V.P. Levashev, Nucl. Phys. **A343** (1980) 317

[6] A.C. Fonseca, Phys. Rev. **C30** (1984) 35

[7] V.B. Belyaev, V.V. Pupyshev, Yad. Fiz. **35** (1982) 905

[8] F. Ciesielski, J. Carbonell, C. Gignoux, Preprint ISN 96.18, Grenoble, 1996

[9] V.S. Vasilevsky *et al.,* Yad. Fiz. **48** (1987) 345

[10] V.P. Alfimenkov *et al.,* Yad. Fiz. **33** (1981) 891

[11] G. Kobschal *et al.,* Nucl. Phys. **99** (1955) 1405